

PROBLEM SET 4
ECON 3102-002 Intermediate Macroeconomics
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Due Date: Wednesday April 26 (in class)

1 CE in a Two-Period Model with investment (50 points)

Consider a two-period economy model with a representative consumer who has utility function:

$$U(c, c', l, l') = \frac{c^{1-\sigma}}{1-\sigma} + \frac{l^{1-\sigma}}{1-\sigma} + \beta \frac{c'^{1-\sigma}}{1-\sigma} + \beta \frac{l'^{1-\sigma}}{1-\sigma}$$

The consumer works and consumes in each period, and is able to save or borrow at interest rate r . He has h available hours each period to divide between leisure and work. He is the owner of the representative firm, so he receives the profits from the firm in each period (π, π') .

There is a government that imposes a proportional labor income tax on the two periods τ, τ' . The revenue from these taxes finance government spending on each period denoted by G and G' . The government can also borrow or lend at interest rate r .

Finally, there is a firm with production function $zK^\alpha(N^d)^{1-\alpha}$ that produces output in each period hiring labor and using capital. Capital in the first period is given, K . The firm can choose how much to invest in future period capital K' . Capital depreciates at rate d each period. At the end of the last period, after production, the firm can sell the undepreciated capital $(1-d)K'$ and distribute the proceeds as profits to the consumer.

1. Define a Competitive Equilibrium for this economy. (10 points)
2. Characterize the CE with a system of 9 equations. Clearly state what variables can be pinned down and which cannot. (10 points)
3. Set up the Social Planner's Problem associated to this economy. (10 points)
4. Characterize the Social Planner's Problem. (10 points)
5. Is the CE Pareto Optimal? Prove it. (10 points)

2 Consumer's Problem with Limited Commitment (50 points)

Consider a consumer who lives for two periods, and cares about consumption and leisure in each period. His utility function is given by $U(c, c', l, l') = \log c + \log c' + \log l + \log l'$. In each period, the consumer has h available hours to divide between leisure and work. For each hour worked, he receives a wage w in the first period, or w' in the second. The consumer can save or borrow at interest rate r , which he takes as given. The consumer also pays lump-sum taxes to the government each period t, t' .

1. Suppose this consumer is able to commit to his promises. Set up the consumer's problem and characterize it. (10 points)
2. Let $w = 4, w' = 5, h = 1, r = 0, t = 2, t' = 1$. Find values for (c, c', l, l') . Is the consumer borrowing or saving? That is, find s for this case. (10 points)

3. Suppose now that the consumer lacks commitment, so that he cannot faithfully keep his promises of repayment. Derive a collateral constraint and set up the problem of this consumer as a constrained maximization problem. (There is no collateral in this case) (5 points)
4. Using the parameter values from section 1, find values for (c, c', l, l') . Is the consumer saving or borrowing? What is s now? (Hint: you should assume that the collateral constraint binds, so the lagrange multiplier on the collateral constraint $\lambda_2 > 0$) (5 points)
5. Now suppose the consumer owns an illiquid asset H , that cannot be sold in the first period but it can be sold in the second period at a price p . This asset can be used as collateral in order to borrow. Set up the consumer's problem in this case and characterize it with a system of 5 equations and 5 unknowns $(c, c', l, l', \lambda_2)$, where λ_2 is the lagrange multiplier on the collateral constraint. (10 points)
6. Suppose the collateral constraint is binding ($\lambda_2 > 0$). Using the parameter values of section 2, and given $p = 0.5, H = 1$, find values for (c, c', l, l') . What is s now? (10 points)